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CBCS/ SEMESTER SYSTEM

COURSE-V, LINEAR ALGEBRA

MATHEMATICS MODEL PAPER

Time:3Hrs Max.Marks:60M

SECTION - A

Answer any FIVE questions 5x4=20M

Each question carries 4 marks

1. Show that W= { (x, y, o)/ x, y ∈ F} is a subspace of $V_3(F)$.
2. If S is a subset of a vector space V(F), then show that L(S) is a subspace of V.
3. Show that the vectors (1,2,1), (2,1,0), (1,-1,2) form a basis for R^3.
4. Describe explicitly the linear transformation T: $R^2 \to R^2$ such that T(2,3) = (4,5) and T(1,0)= (0,0).
5. If T is a linear transformation from a vector space of U(F) into V(F), then N(T), the null space of T is a subspace of U.
6. Find the rank of the matrix A= $\begin{pmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{pmatrix}$

7.If $\alpha = (x_1, x_2, \ldots x_n)$ and $\beta = (y_1, y_2, \ldots . y_n)$ are vectors in R^n vector space then prove that $(\alpha, \beta) = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n$ defines an inner product in R^n.

8. Show that $\left\{ \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} \right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3} \right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right) \right\}$ form orthonormal set in an Inner product space $R_3(R)$.

SECTION – B

Answer all questions , Each question carries eight marks. 5 x 8M = 40M

9.(a) Let V(F) be a vector space, If $\alpha_1, \alpha_2, \ldots \alpha_n$ are non zero vectors of V then either they are linearly independent or some α_k, $2 \leq k \leq n$ is a linear combination of the preceding ones $\alpha_1, \alpha_2, \ldots \alpha_{k-1}$.

OR

(b). Let V(F) be a vector space and W be a non-empty subset of V. Prove that the necessary and sufficient condition for W to be a subspace of V is

$a\alpha + b\beta \in W$ for all $a, b \in F$ and $\alpha, \beta \in W$

10. (a) If W is a subspace of a finite dimensional vector space V(F) then $\dim\left(\frac{V}{W}\right) = \dim V - \dim W$.

OR

(b) Prove that every linearly independent subset S={ $\alpha_1, \alpha_2, \ldots \alpha_n$ } of a finite dimensional vector space V(F) is either a basis of V or it can be extended to form a basis of V.

11. (a) If T is a linear transformation from a vector space U(F) into a vector space V(F) and U is a finite dimensional then prove that rank(T)+ nullity (T) = dimU.

OR

(b) If T is a linear transformation from a vector space U(F) into V(F), then prove that R(T) the range of T is a subspace of V.

12.(a) Find the characteristic roots and the corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$

OR

(b). Show that every square matrix satisfies its characteristic equation.

13.(a) State and Cauchy-schwartz inequality in an inner product space V(F).

OR

(b). (i) State and prove triangle inequality in an inner product space V(F).

(ii) State and prove parallelogram law in an inner product space V(F).